**Optimal Influence strategies in oligopolistic markets**

1. ***Assumptions-Description of the Economy and the Network***

* Oligopolistic product market (price competition)
* Two firms: 
* Two consumers: 
* Set of nodes in the network: 
* Heterogeneous product
* Quantity produced by firm j: 
* Quantity produced by firm j and consumed by consumer i: 
* Production cost function: , where 
* Total quantity: 

with  and 

* Demand functions:

For consumer 1

Demand function for product A: 

Demand function for product B: 

For consumer 2

Demand function for product A: 

Demand function for product B: 

where *μ*/*κ* is the demand’s responsiveness to the competitor’s price/quality with *μ<1* and *κ<1*

* Social interactions matrix



* Let , i.e firms don’t communicate with each other and let



* Row stochasticity:



* Social network structure:

 

  

  

 

* Restrictions:















* Beliefs for j product of consumer i: 

For T=0,  for A product

 for B product

* Assumptions for believes

Initial beliefs:

, i.e firm A/B believes that her product is the best in the period 0.

, i.e. firm A/B believes that the competitive product is the worst in the period 0.

Updating rule:



Limiting belief:



i.e 

* Firms can make an investment to increase their direct influence on consumer i’s beliefs , with total investment cost function  (with *γ*>0) where i.e. 
* Firms’ profits:

 i.e

,



* Time structure of the model:
* T=1: firms choose simultaneously the level of investments to influence consumers 1 and 2.
* T=2: firms choose simultaneously the prices for each product.
* T=3: Agents in the network repeatedly communicate to update their beliefs and reach a limiting vector of beliefs for product A and  for product B. Then, consumers make their purchasing decisions according to their demand.

1. ***Derivation of Equilibrium***

* T=3: computation of agents’ limiting belief
* Strongly connected and aperiodic matrix imply belief consensus in the limit for each product.
* We compute Influence vector for A and B product, i.e.

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as the left-hand eigenvector associated with the unit eigenvalue of the matrix 

* Computation of limiting believes:

, i.e

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* T=2: firms choose pricing strategy
* Firm A:

max 





Best response function of firm A:

* Firm B:







Best response function of firm B: 

* **Optimal pricing for product A**: 



* **Optimal pricing for product B**: 



* T=1: firms choose simultaneously the level of investments to influence consumers 1 and 2.
* We compute the optimal quantities for product A and product B replacing optimal prices in denand functions





* Firms choose  and  so as to mazimize profits:



